FINITE ELEMENT ANALYSIS OF THE ACOUSTIC FIELD IN AN AXISYMMETRIC DUCT WITH FLOW USING THE CONVECTED WAVE EQUATION

By

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1981

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
MASTER OF SCIENCE
May, 1983

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ACKNOWLEDGMENTS

I owed the successful completion of this degree to many people. I would like to appreciate to Dr. Cline T. Young his friendly suggestions and encouragement, to Dr. J. K. Good and Dr. Richard L. Lowery their understanding. In particular, I would like to express my deep appreciation to Dr. James J. Allen, my thesis adviser, for his patient advise, wise guidance and encouragement during my graduate study.

I am also grateful to Soon Man Han for her assistance in preparing the manuscript.

I am indebted to my parents, Mr. and Mrs. Jong Ho Kwon, who gave me the opportunity for a higher education. I also wish to express very special thanks to my wife, Soon Ja, for her constant patience, encouragement, and sacrifice which made my graduate study successful. I would once again like to thank everybody who helped me to finish this degree successfully.

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NOMENCLATURE

a	-	Radius of a circular duct
A	-	Area
c	-	Sound speed
Cm	-	Conformability of order m
J_m	-	The first kind of Bessel function of order m
$J_m'(*)$	-	The first orderdderivative of J_m with respect to *
k	-	Loading wave number, ω/c
k_{mn}	-	Cut-off wave number of a duct
L	-	Boundary line
m,n	-	Mode indices
M	-	Mach number, U/c
n	-	Outer normal direction of a boundary surface
Ni	-	Shape function
P	-	Acoustic pressure
P_c	-	Acoustic pressure at the center line of a duct
P_i	-	Acoustic pressure at the ith nodal point
P_{inci}	-	Input acoustic pressure given at the input boundary
P_o	-	Mean pressure
R	-	Residual
Sw, Si	-	Boundary surfaces of a duct
t	-	Time
u	-	Particle velocity in an axial direction
U	-	Mean fluid velocity in an axial direction

v	-	Particle velocity in a radial direction
V	-	Mean fluid velocity in a radial direction
$\overline{V}$	-	Volume of system
w	-	Particle velocity in a circumferential direction
W_i	-	Weight function
r, θ, z	-	A set of cyclindrical coordinate
x, θ, y	-	Other set of cyclindrical coordinate
z	-	Normal specific acoustic impedance
α	-	Direction cosine of wave propagation with respect to an axial direction
ρ ,ρ	-	Specific density of the fluid medium
ω	-	Angular frequency
∇^2	-	Laplacian operator
$\lfloor \; \rfloor$	-	Row matrix

CHAPTER I

INTRODUCTION

Background

In order to give a person a better environment, the acoustic environment needs to be controlled. A frequently encountered situation is acoustic propagation in ducts. Thus, duct acoustic problems are often encountered to control noise, for example, in ventilation ducts, mufflers of passenger cars, and nuclear reactors. Most duct acoustic problems involve fluid flow inside a duct. Many parameters, however, such as fluid flow in a duct, complex geometry of a duct and sound radiation from the open end of a duct make duct acoustic problems complicated. Consequently it is hard to obtain the analytic solutions for such complicated problems so that many different solution techniques such as approximate methods, finite difference method, and finite element method have been developed.

The approximate method usually involves very limiting assumptions which restrict its usefulness. The finite difference method gives good numerical results but it is difficult to be applied to complex geometry of ducts. Thus, the finite element analysis will be adopted in this paper.

Literature Survey

During past decades many researchers have worked on duct acoustic problems. They used analytic methods, numerical methods or both of them to solve the problems. First of all, the analytic results will be reviewed and later the numerical results will be reviewed.

The acoustic problem not including fluid flow was solved by Morse [1]. He developed the theory about sound attenuation in a duct with no flow. After that, Lansing and Zorumski [2] also dealt with the duct of uniform area with lining variations but no flow, and Cho and Ingard [3] studied higher order mode propagation in a nonuniform circular duct without mean flow. They derived an approximate wave equation on the assumptions that the duct has very slow change of cross-section and given mode continues through the duct without mode change. They used "Circular Cosh Duct" to develope their theory.

The acoustic problem including fluid flow was worked on by Eversman and Astley [4,5]. They studied the acoustic problem in a non-uniform duct with flow, and they considered a high speed subsonic compressible flow inside a duct. They also used two methods to attack this problem; the method of weighted residuals and the finite element method. The comparison between the results obtained using two methods was also given in those papers and two results were in good agreement. The plane wave in ducts with one dimensional flow was also dealt by King and Karamcheti [6].

Nayfeh [7] studied sound propagation through non-uniform ducts with and without flow. He reviewed the state of art concerning methods dealing with non-uniform duct acoustic problems, and the reviewed methods were purely numerical methods, quasi-one-dimensional approximations, solutions for slowly varying cross section, solutions for weak wall undulations, approximation of the duct by a series of stepped uniform cross sections, variational methods and solutions for the mode envelops.

Callegari and Myers [8] studied the propagation of sound inside a converging-diverging duct containing a quasi-one-dimensional steady flow with a high subsonic throat Mach number, and they concluded that the linearized acoustic theory at the throat of the duct was invalid. Nayfeh, Shaker and Kaiser [9] developed an acoustic theory to determine the sound transmission and attenuation in a lined and non-uniform duct including compressible, sheared, mean flow. They used the wave envelope technique to solve for the envelopes of the quasi-parallel acoustic modes that exist in the duct instead of solving for the actual wave. However, this technique was proved to be not suitable near cut-off frequency.

The analytical approaches mentioned above have many limitations in solving the acoustic problem. Hence, lots of numerical techniques have been developed. Among the numerical methods, the finite element method has become most popular because of its general and diverse applicability. The first attack to the acoustic problems using finite element technique was initiated by Gladwell [10] who used a one-dimensional element. Craggs [11] predicted the sound transmission loss from the engine compartment and the passenger space of a passenger car using three-dimensional box elements. Abrahamson [12] determined the mathematical model used for obtaining optimum acoustic liners in aeroengine ducts using finite elements. He also developed an accurate mathematical model for sound propagation in axisymmetric aircraft engine ducts with compressible mean flow and the model is based on the perturbation theory of fluid dynamics [13].

Baumeister et al. [14] used both finite element method and finite difference method for the study of sound propagation without flow in a rectangular duct with a converging-diverging area variation. They also performed experimental study for this problem. The results obtained using three different methods were in good agreement. Young [15] used finite element method to compute the sound attenuation performance of mufflers for internal combustion engine, and he stressed the finite element method over other numerical approaches because the former could be applied to the muffler with completely general geometry and complex boundary conditions which included not only displacement but also pres-ure boundary conditions. Baumeister [16] reviewed both finite element and finite difference methods for the sound propagation in straight and variable area ducts including flow. Sigman et al. [17] studied the acoustic properties of turbofan inlets containing high subsonic Mach number flow using finite element method and Galerkin's method. Ling [18] also exploited two-dimensional isoparametric elements to solve several duct acoustic problems.

Acoustic Domain Equations

In order to solve the duct acoustic problem, the finite element analysis can be based on several different domain equations. The type of shape functions and required conformability depend on the domain equation to be used.

Abrahamson [13] derived four governing equations for the linearized acoustic motion in a nonuniform axisymmetric duct using basic fluid dynamic equations such as conservation of momentum, mass and energy equations and equation of state. The derived equations are

$$\omega u + (v + \frac{PV}{\rho o c^2}) \frac{U}{r} + V \frac{u}{r} + (u + \frac{PU}{\rho o c^2}) \frac{U}{z} + U \frac{u}{z} + \frac{1}{\rho_o} \frac{P}{z} = 0 \qquad (1.1)$$

$$\omega v + (v + \frac{PV}{\rho o c^2}) \frac{V}{r} + V \frac{v}{r} + (u + \frac{PU}{\rho o c^2}) \frac{V}{z} + U \frac{v}{z} + \frac{1}{\rho o} \frac{P}{r} = 0 \qquad (1.2)$$

$$\omega w + v(\frac{\partial w}{\partial r} + \frac{1}{r} w) + U \frac{\partial w}{\partial z} + \frac{m}{r} p = 0 \qquad (1.3)$$

$$\frac{\omega}{c^2} + \frac{U}{c^2}(\frac{\partial P}{\partial z} - \frac{2P}{c} \frac{\partial c}{\partial z}) + \frac{V}{c^2}(\frac{\partial P}{\partial r} - \frac{P}{c} \frac{\partial c}{\partial r}) + U \frac{\partial P}{\partial z} + v \frac{\partial P}{\partial r} +$$

$$\rho (\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} + \frac{m}{r} w + \frac{v}{r}) + \frac{P}{c^2}(\frac{\partial V}{\partial z} + \frac{\partial V}{\partial r} + \frac{V}{r}) = 0 \qquad (1.4)$$

To apply the finite element method to Equations (1.1) through (1.4), four types of shape functions are needed for the variables u,v,w, and P. In addition, according to the conformability condition the equation whose order is equal to 2m requires m-1 conformability. The notation Cm-1 is used to indicate m-1 conformability. Thus, Equations (1.1) through (1.4) require Co conformability because they are first order equations. Astley and Eversman [5] used the same equation as above but they neglected the velocity in Z-direction.

Pridmore-Brown [19] combined all basic fluid dynamic equations into single equation with the assumption of tranversly one-dimensional sheared flow. The derived equation is

$$\frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} = (1 - M^2) \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \frac{2M}{c} \frac{\partial^2 P}{\partial x \partial t} + 2\rho c \frac{dM}{dy} \frac{\partial v}{\partial x} = 0 \qquad (1.5)$$

Savkar [20] deduced the same equation as Equation (1.5) but he adopted the momentum equation for the last term of right hand side of Equation (1.5). The resultant equation is

$$\frac{D}{Dt}\left(\frac{D^2P}{Dt^2} - c^2\nabla^2P\right) + 2c^2\,\frac{dU}{dy}\,\frac{\partial^2P}{\partial x\partial y} = 0 \tag{1.6}$$

which requires C_1 conformability. If velocity U is assumed to be constant with respect to y, that is d/dy = 0, then Equation (1.6) is transformed into the convected wave equation

$$\frac{D^2P}{Dt^2} - c^2\nabla^2P = 0 \tag{1.7}$$

Here the substantial derivative D/Dt is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U\,\frac{\partial}{\partial x} \tag{1.8}$$

with one-dimensional flow. Moreover, with an assumption that the pressure is a time harmonic function such as

$$P(t,x) = P'(x)\,e^{i\omega t} \tag{1.9}$$

Equation (1.7) is transformed into

$$\nabla^2P + k^2P - 2i\omega\,\frac{U}{c^2}\,\frac{\partial P}{\partial x} - \frac{U^2}{c^2}\,\frac{\partial^2P}{\partial x^2} = 0 \tag{1.10}$$

This equation was used by Ling [18]. In this equation only one shape function for the unknown parameter, P, is needed and Co conformability is required to make the finite element solution converge as the element mesh becomes smaller and smaller.

If two-dimensional flow is considered, the substantial derivative becomes

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y} \qquad (1.11)$$

Therefore, using Equation (1.7) and (1.11), the conveted wave equation can be expressed as

$$\nabla^2 P + (k^2 - \frac{m^2}{y^2})P - i\frac{2U}{c^2}\frac{\partial P}{\partial x} - i\frac{2V}{c^2}\frac{\partial P}{\partial y} - \frac{2UV}{c^2}\frac{\partial^2 P}{\partial x \partial y}$$

$$- \frac{U^2}{c^2}\frac{\partial^2 P}{\partial x^2} - \frac{V^2}{c^2}\frac{\partial^2 P}{\partial y^2} = 0 \qquad (1.12)$$

The above equation also requires one shape function for P and Co conformability and this equation will be used throughout this paper. The significant assumption in deriving Equation (1.12) is that the mean velocity remains constant; however, when used in a finite element formulation the velocity will vary from element to element. If a smaller element mesh is used in finite element analysis, the assumption will approximate a velocity varying throughout the system domain.

Objective

The objective of this thesis is to analyze the acoustic field in an axisymmetric duct with or without flow using the finite element method. This method will be applied to several example problems. First of all, the finite element method combined with the convected wave equation is applied to simple problems and each finite element solution is compared with the analytic solution or the experimental result to verify the accuracy of the finite element solution.

Next, the finite element method combined with the convected wave equation is applied to the duct acoustic problem containing shear flow in a duct to check if this method can be used to solve the problem involving shear flow. As more complex example, the finite element method is applied to find the sound propagation in a converging-diverging duct with two-dimensional flow.

Through this research an axisymmetric circular duct with hard wall is used, and the domain equation to be used can be rewritten as

$$\nabla^2 P + \left(k^2 - \frac{m^2}{y^2}\right) P - i\,\frac{2U}{c^2}\frac{\partial P}{\partial x} - i\,\frac{2V}{c^2}\frac{\partial P}{\partial y} - \frac{2UV}{c^2}\frac{\partial^2 P}{\partial x \partial y}$$

$$- \frac{U^2}{c^2}\frac{\partial^2 P}{\partial x^2} - \frac{V^2}{c^2}\frac{\partial^2 P}{\partial y^2} \doteq 0 \qquad (1.13)$$

The boundary conditions, which are derived in Appendix A, are

$$i\rho\omega\,\frac{P}{z} + \frac{\rho U}{z}\frac{\partial P}{\partial x} + \frac{V}{z}\frac{\partial P}{\partial y} = -\frac{\partial P}{\partial n} \qquad \text{on Sw} \qquad (1.14)$$

$$\frac{\partial P}{\partial n} = f(x,y) \qquad \text{on Si} \qquad (1.15)$$

where f(x,y) is a given input boundary condition, and Sw and Si are shown in Figure 1.

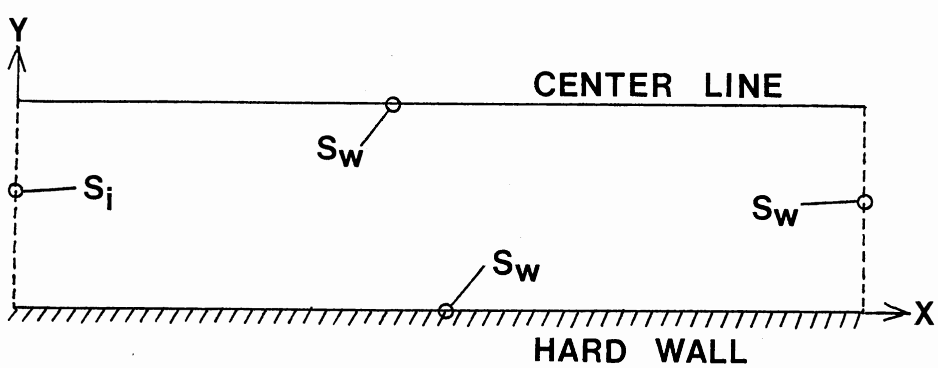


Figure 1. Idealization of Acoustic System

CHAPTER II

DEVELOPMENT OF GALERKIN FINITE ELEMENT METHOD

Galerkin Finite Element Formulation For Acoustic Field Inside Circular Ducts

As mentioned in the previous chapter, the domain equation

$$\nabla^2 P + \left(k^2 - \frac{m^2}{y^2}\right) P - i\,\frac{2U}{c^2}\frac{\partial P}{\partial x} - i\,\frac{2V}{c^2}\frac{\partial P}{\partial y} - \frac{2UV}{c^2}\frac{\partial^2 P}{\partial x \partial y}$$

$$- \frac{U^2}{c^2}\frac{\partial^2 P}{\partial x^2} - \frac{V^2}{c^2}\frac{\partial^2 P}{\partial y^2} = 0 \qquad (2.1)$$

will be used to formulate the acoustic-flow problem in an axisymmetric duct. In addition, the Galerkin method will be adopted to formulate the finite element equation corresponding to Equation (2.1).

Galerkin method is one of the weighted residual methods which include collocation, least squares, least square collocation, and moment methods. According to Cook [21] the methods differ in how the weight function Wi is defined, and the general procedure is to establish a tentative group of solutions and decide the best solution of the group based on a given criterion. The criterion for weighted residual methods can be written as

$$\int_V Wi \; R \; d\overline{V} = 0 \qquad (2.2)$$

where R is residual of the governing equation.

In Galerkin's method, the shape function Ni can be used as the weight function Wi and then Equation (2.2) may be written as

$$\int_{\overline{V}} Ni\ R\ d\overline{V} = 0 \tag{2.3}$$

Next, consider the finitely subdivided domain as shown in Figure 2. On the finite subdomain the acoustic pressure P can be expressed as

$$P = \sum_{i=1}^{K} Ni\ Pi \tag{2.4}$$

where K is the number of nodal points on the subdomain, V^e. Using a matrix notation, Equation (2.4) may be rewritten as

$$P = \lfloor N \rfloor \ \{P^e\} \tag{2.5}$$

To obtain the residual R^e on each subdomain Equation (2.5) is inserted into Equation (2.1). Then

$$R^e = (\lfloor \nabla^2 N \rfloor + (k^2 - \frac{m^2}{y^2})\lfloor N \rfloor - i\frac{2U\omega}{c^2}\lfloor \frac{\partial N}{\partial x} \rfloor - i\frac{2V\omega}{c^2}\lfloor \frac{\partial N}{\partial x} \rfloor$$

$$- \frac{2UV}{c^2}\lfloor \frac{\partial^2 P}{\partial x \partial y} \rfloor - \frac{U^2}{c^2}\lfloor \frac{\partial^2 N}{\partial x^2} \rfloor - \frac{V^2}{c^2}\lfloor \frac{\partial^2 N}{\partial y^2} \rfloor \ ) \ \{P^e\} \tag{2.6}$$

Equation (2.6) is a trial solution, so the best solution can be selected using the criterion Equation (2.3). Inserting Equation (2.6) into Equation (2.3) yields

$$\{I^e\} = \int_{V^e} \{N\}\lfloor \nabla^2 N \rfloor \ dV\{P^e\} + \int_{V^e} k^2\{N\}\lfloor N \rfloor \ dV\{P^e\}$$

$$- \ m^2 \int_{V^e} \frac{1}{y^2}\ \{N\}\lfloor N \rfloor \ dV\{P^e\} - 2i \int_{V^e} \frac{U}{c^2}\ \{N\}\lfloor \frac{\partial N}{\partial x} \rfloor \ dV\{P^e\}$$

$$- 2i\int_{V^e} \frac{V}{c^2} \{N\}\lfloor\frac{\partial N}{\partial y}\rfloor\, dV\{P^e\} - 2\int_{V^e} \frac{UV}{c^2} \{N\}\lfloor\frac{\partial^2 N}{\partial x\partial y}\rfloor\, dV\{P^e\}$$

$$-\int_{V^e} \frac{U^2}{c^2} \{N\}\lfloor\frac{\partial^2 N}{\partial x^2}\rfloor\, dV\{P^e\} - \int_{V^e} \frac{V^2}{c^2} \{N\}\lfloor\frac{\partial^2 N}{\partial y^2}\rfloor\, dV\{P^e\} \qquad (2.7)$$

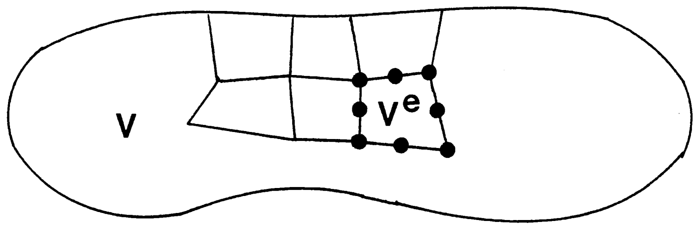


Figure 2. System Domain

The Green's theorem is applied to the first, sixth, seventh and eighth terms to reduce the order of Equation (2.7). The first term is transformed using the Green's theorem into the following form

$$\int_{S^e} \{N\} \lfloor\frac{\partial N}{\partial n}\rfloor\, dS\{P^e\} - \int_{V^e} \{\nabla N\}\lfloor\nabla N\rfloor\, dV\{P^e\} \qquad (2.8)$$

Here, to perform a surface integral of the first term in Equation (2.8) the boundary conditions must be introduced. Rewriting the boundary conditions here yields

$$i\rho\omega \frac{P}{z} + \frac{U}{z}\frac{\partial P}{\partial x} + \frac{V}{z}\frac{\partial P}{\partial y} = - \frac{\partial P}{\partial n} \qquad \text{on } S_w \qquad (2.9)$$

$$\frac{\partial P}{\partial n} = f(x,y) \qquad \text{on } S_i \qquad (2.10)$$

where S_w and S_i are shown in Figure 1 and $f(x,y)$ is a given input boundary condition, and the subscript o for ρ is omitted. Applying Equations

(2.9) and (2.10) to Equation (2.8) yields

$$\int_{S_i^e} \{N\}\, f(x,y)\, dS - i\omega \int_{S_W^e} \frac{P}{z} \{N\}\lfloor N \rfloor\, dS\{P^e\}$$

$$- \int_{S_W^e} \frac{\rho U}{z} \{N\}\lfloor \frac{\partial N}{\partial x} \rfloor\, dS\{P^e\} - \int_{S_W^e} \frac{\rho V}{z} \{N\}\lfloor \frac{\partial N}{\partial y} \rfloor\, dS\{P^e\}$$

$$- \int_{V^e} \{\nabla N\}\lfloor \nabla N \rfloor\, dV\{P^e\} \tag{2.11}$$

In a similar way, the sixth, seventh, and eighth terms of Equation (2.7) are transformed into the forms, respectively,

$$\int_{S_W^e} \frac{2UV}{c^2} \{N\} \lfloor \frac{\partial N}{\partial y} \rfloor (\bar{i}_y \cdot \bar{n})\, dS\{P^e\} - \int_{V^e} \frac{2UV}{c^2} \{\frac{\partial N}{\partial x}\} \lfloor \frac{\partial N}{\partial y} \rfloor\, dV\{P^e\} \tag{2.12}$$

$$\int_{S_i^e} \frac{U^2}{c^2} \{N\}\, f(x,y)\, dS + \int_{S_W^e} \frac{U^2}{c^2} \{N\}\lfloor \frac{\partial N}{\partial x} \rfloor (\bar{i}_x \cdot \bar{n})\, dS\{P^e\}$$

$$\int_{V^e} \frac{U^2}{c^2} \{\frac{\partial N}{\partial x}\} \lfloor \frac{\partial N}{\partial x} \rfloor\, dV\{P^e\} \tag{2.13}$$

$$\int_{S_W^e} \frac{V^2}{c^2} \{N\}\lfloor \frac{\partial N}{\partial y} \rfloor (\bar{i}_y \cdot \bar{n})\, dS\{P^e\} - \int_{V^e} \frac{V^2}{c^2} \{\frac{\partial N}{\partial y}\} \lfloor \frac{\partial N}{\partial y} \rfloor\, dV\{P^e\} \tag{2.14}$$

Substituting Equations (2.11), (2.12), (2.13) and (2.14) for the first, sixth, seventh and eighth terms of Equation (2.7) and rearranging the result, we obtain the resultant equation

$$(\omega^2[M^e] - i\omega([CI^e] + 2[DI^e] + 2[DJ^e]) - [IU^e] - [IV^e]$$

$$-[K^e] - [L^e] - [E^e] + [F^e] - [GI^e] + [HI^e] - [GJ^e] + [HJ^e])\{P^e\}$$

$$- [R^e] - [Q^e] = \{I^e\} \tag{2.15}$$

where

$$[M^e] = \int_{V^e} \frac{1}{c^2} \{N\}\lfloor N \rfloor \, dV \qquad \text{Acoustic Inertia} \tag{2.16}$$

$$[CI^e] = \int_{S_w{}^e} \frac{\rho}{z} \{N\} \lfloor N \rfloor \, dS \qquad \text{Boundary Impedance Damping} \tag{2.17}$$

$$[DI^e] = \int_{V^e} \frac{U}{c^2} \{N\}\lfloor \frac{\partial N}{\partial x} \rfloor \, dV \qquad \text{Horizontal Flow Damping} \tag{2.18}$$

$$[DJ^e] = \int_{V^e} \frac{V}{c^2} \{N\} \lfloor \frac{\partial N}{\partial x} \rfloor dV \qquad \text{Vertical Flow Damping} \tag{2.19}$$

$$[IU^e] = \int_{S_w{}^e} \frac{\rho U}{z} \{N\} \lfloor \frac{\partial N}{\partial x} \rfloor \, dS \qquad \text{Horizontal Flow Effect on Boundary Impedance} \tag{2.20}$$

$$[IV^e] = \int_{S_w{}^e} \frac{\rho V}{z} \{N\}\lfloor \frac{\partial N}{\partial y} \rfloor \, dS \qquad \text{Vertical Flow Effect on Boundary Impedance} \tag{2.21}$$

$$[K^e] = \int_{V^e} \{\nabla N\}\lfloor \nabla N \rfloor \, dV \qquad \text{Acoustic Stiffness} \tag{2.22}$$

$$[L^e] = \int_{V^e} \frac{m^2}{y^2}\{N\}\lfloor N \rfloor \, dV \qquad \text{Higher Mode Stiffness} \tag{2.23}$$

$$[E^e] = \int_{S_w{}^e} \frac{2UV}{c^2} \{N\}\lfloor N \rfloor \, (\bar{i}_y \cdot \bar{n}) \, dS \qquad \text{Coupled Flow Effect on Boundary Stiffness} \tag{2.24}$$

$$[F^e] = \int_{V^e} \frac{2UV}{c^2} \{\frac{\partial N}{\partial x}\} \lfloor \frac{\partial N}{\partial y} \rfloor \, dV \qquad \text{Coupled Flow Convection Stiffness} \tag{2.25}$$

$$[GI^e] = \int_{S_w{}^e} \frac{U^2}{c^2} \{N\} \lfloor \frac{\partial N}{\partial x} \rfloor \, (\bar{i}_x \cdot \bar{n}) \, dS \qquad \text{Horizontal Flow Effect on Boundary Stiffness} \tag{2.26}$$

$$[HI^e] = \int_{V^e} \frac{U^2}{c^2} \{\frac{\partial N}{\partial x}\} \lfloor \frac{\partial N}{\partial x} \rfloor \, dV$$ Horizontal Flow Convection Stiffness (2.27)

$$[GJ^e] = \int_{S_w^e} \frac{V^2}{c^2} \{N\} \lfloor \frac{\partial N}{\partial y} \rfloor (\bar{i}_y \cdot \bar{n}) \, dS$$ Vertical Flow Effect on Boundary Stiffness (2.28)

$$[HJ^e] = \int_{V^e} \frac{V^2}{c^2} \{\frac{\partial N}{\partial y}\} \lfloor \frac{\partial N}{\partial y} \rfloor \, dV$$ Vertical Flow Convection Stiffness (2.29)

$$[R^e] = \int_{S_i^e} \{N\} \, f(x,y) \, dS$$ Particle Velocity Boundary Input (2.30)

$$[Q^e] = \int_{S_i^e} \frac{U^2}{c^2} \{N\} \, f(x,y) \, dS$$ Convection Effect on Input (2.31)

Neglecting the terms which contain the vertical velocity V, we will obtain the same equation as was obtained by Ling [18].

If there is no flow in a duct, only five terms, $[M^e]$, $[CI^e]$, $[K^e]$, $[L^e]$, and $[R^e]$ of sixteen terms will be left in Equation (2.15). If we also assume one-dimensional flow inside a duct, there will be ten terms left.

As mentioned before, there has been an assumption in deriving Equation (2.15) that the velocity of an airflow in a finite element mesh is constant in each direction. Thus, the average value of the velocities in a mesh can be used in each direction.

Equation (2.15) may be rewritten as

$$[T^e]\{P^e\} - \{S^e\} = \{I^e\} \tag{2.32}$$

where $[T^e]$ and $\{S^e\}$ are the sums of all square matrices and column vectors, respectively, in Equation (2.15). Assembling Equation (2.32) over whole elements yields

$$[T]\ \{P\} = \{S\} \tag{2.33}$$

where [T] and {S} mean the sums of all element square matrices and element column vectors, respectively.

Equation (3.33) is the final equation to be solved in order to compute the acoustic pressure {P} in a duct. This is a linear algebraic matrix equation, and several numerical solution methods can be utilized to obtain the acoustic pressure {P}.

Isoparametric Element and Numerical Integration

Isoparametric Element

Many different kinds of finite elements can be used in deriving the Galerkin finite element equation. However, the isoparametric element is particulary suitable for the curved boundary element. Therefore, we will adopt the isoparametric element for the formulation of finite element equation.

The development of isoparametric elements is given in many References [21,22]. Baumeister [16] presented a summary of finite element configurations used in acoustic applications. In this thesis, two-dimensional quadratic isoparametric elements will be used as shown in Figure 3.

The Serendipity shape functions for a two-dimensional quadratic isoparametric element are expressed as

$$N1 = -(1-r)(1-s)(r+s+1)/4 \tag{2.34}$$

$$N2 = (1-r)(1+r)(1-s)/2 \tag{2.35}$$

$$N3 = (1+r)(1-s)(r-s-1)/4 \tag{2.36}$$

$$N4 = (1+r)(1+s)(1+r)/2 \tag{2.37}$$

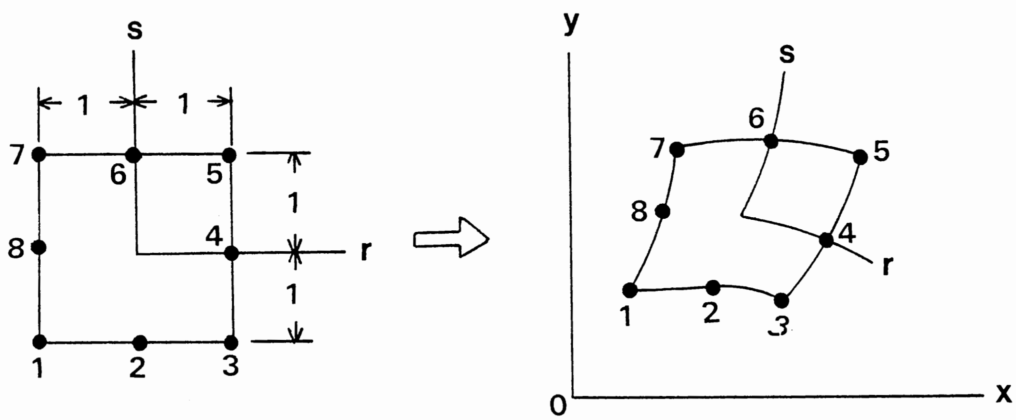


Figure 3. Two-dimensional Quadratic Isoparametic Element and Its Mapping

$$N5 = (1+r)(1-s)(r+s-1)/4 \tag{2.38}$$

$$N6 = (1-r)(1+r)(1+s)/2 \tag{2.39}$$

$$N7 = -(1-r)(1+s)(r-s+1)/4 \tag{2.40}$$

$$N8 = (1-r)(1+r)(1-s)/2 \tag{2.41}$$

The shape function Ni has unit value on the ith nodal point and zero value on any other nodal points.

Numerical Integration

It is difficult to obtain a closed form of the integration of Equations (2.16) through (2.31). Therefore, numerical integration methods must be used to compute each terms. Gerald [23] introduced many numerical integration techniques. The Gauss Legendre quadrature integration has been proved most advantageous in finite element work because a polynomial of degree 2n-1 is integrated exactly using n-point Gauss legendre quadrature. In one-dimensional problems, the quadrature formula for g(x) is expressed as

$$\int_{-1}^{1} g(x)\, dx = \sum_{i=1}^{n} W_i\, g(x_i) \tag{2.42}$$

where n is the number of sampling points, and W_i and $g(x_i)$ are the weights and values of the function at the sampling points. The example of sampling points is given in References [21,22, 23].

Because we are dealing with a circular axisymmetric duct, the volume and surface integrations in Equations (2.16) through (2.31) can be replaced by area and line integrations, respectively, by the following relations:

$$d\overline{V} = 2\pi y\, dA \tag{2.43}$$

and

$$dS = 2\pi y \; dL \tag{2.44}$$

When Equations (2.43) and (2.44) are used for the integration of an isoparametric element, the results are

$$\int_{\overline{V}} F \; d\overline{V} = \int_A F \; 2\pi y \; dA = \iint F(r,s) \; 2\pi \lfloor N \rfloor \{Y\} |J| \; dr \; ds \tag{2.45}$$

$$\int_S G \; dS = \int_L G \; 2\pi y \; dL = \int G(r) \; 2\pi \lfloor N \rfloor \{Y\} \; |J| \; dr \tag{2.46}$$

where $\lfloor N \rfloor \{Y\}$ is equal to y and $|J|$ is the determinant of the Jacobian matrix. For two-dimensional problems, the Jacobian matrix, [J], is expressed as

$$[J] = \begin{bmatrix} x,_r & y,_r \\ x,_s & y,_s \end{bmatrix} \tag{2.47}$$

where $x,_r$ indicates the partial derivative of x with respect to r. Thus, the Gauss legendre quadrature can be directly applied to Equations (2.45) and (2.46).

CHAPTER III

APPLICATION OF ACOUSTIC-FLOW FINITE ELEMENT

Several example problems are solved using the finite element formulation which was derived in the previous chapter. The finite element application is made to simple problems to verify the accuracy of the finite element solution and after that more complicated problems.

Sound propagation in a duct including no fluid flow is considered. The propagation of a plane wave and a higher mode wave in a straight duct is solved using the finite element formulation. Here the cut-off frequency is considered for the case of a higher mode wave. Next, the sound propagation of a plane wave in a bottle-like duct with closed end is treated, and the finite element solution is compared with the experimental result.

Later, sound propagation in a duct including a fluid flow is considered as example problems. The propagation of plane wave in a duct with uniform fluid flow is solved, and these results are compared with the analytical solutions. After that, the sound propagation in a duct including a constant gradient fluid flow is treated. In this example the pressure distribution across a duct is investigated.

Finally, as a more complicated example problem, the sound propagation in a converging-diverging duct including two-dimensional fluid flow is studied. In this case, the velocity distribution of fluid inside a converging-diverging duct is obtained using the finite element analysis.

Acoustic Field In Circular Straight Ducts With No Flow

When no flow is present in a duct, Equation (2.1) is reduced to

$$\nabla^2 P + (k^2 - \frac{m^2}{y^2}) P = 0 \qquad (3.1)$$

Moreover, considering a plane wave along a straight duct also reduces Equation (3.1) to

$$\frac{\partial^2 P}{\partial x^2} + k^2 P = 0 \qquad (3.2)$$

where x is the axial direction of a circular straight duct. For this simple problem, the percentage error of the finite element solution to the exact solution versus the number of elements per wavelength is shown in Figure 4, which is called the convergence curve for the no flow case using two-dimensional quadratic isoparametric elements. According to this curve, we can decide how many elements per wavelength are needed to obtain the finite element solution within a certain percentage error when compared with the exact solution. As an example, the finite element solution obtained using eight elements per wavelength is shown in Figure 5 and compared with the analytic solution. Both results are in very good agreement.

The convergence curve for the no flow case using two-dimensional cubic isoparametric elements was given in Reference [18]. When the quadratic and cubic isoparametric elements are compared for this case, three and a half quadratic elements and three cubic elements are needed respectively to obtain the solutions with one percentage error. Even

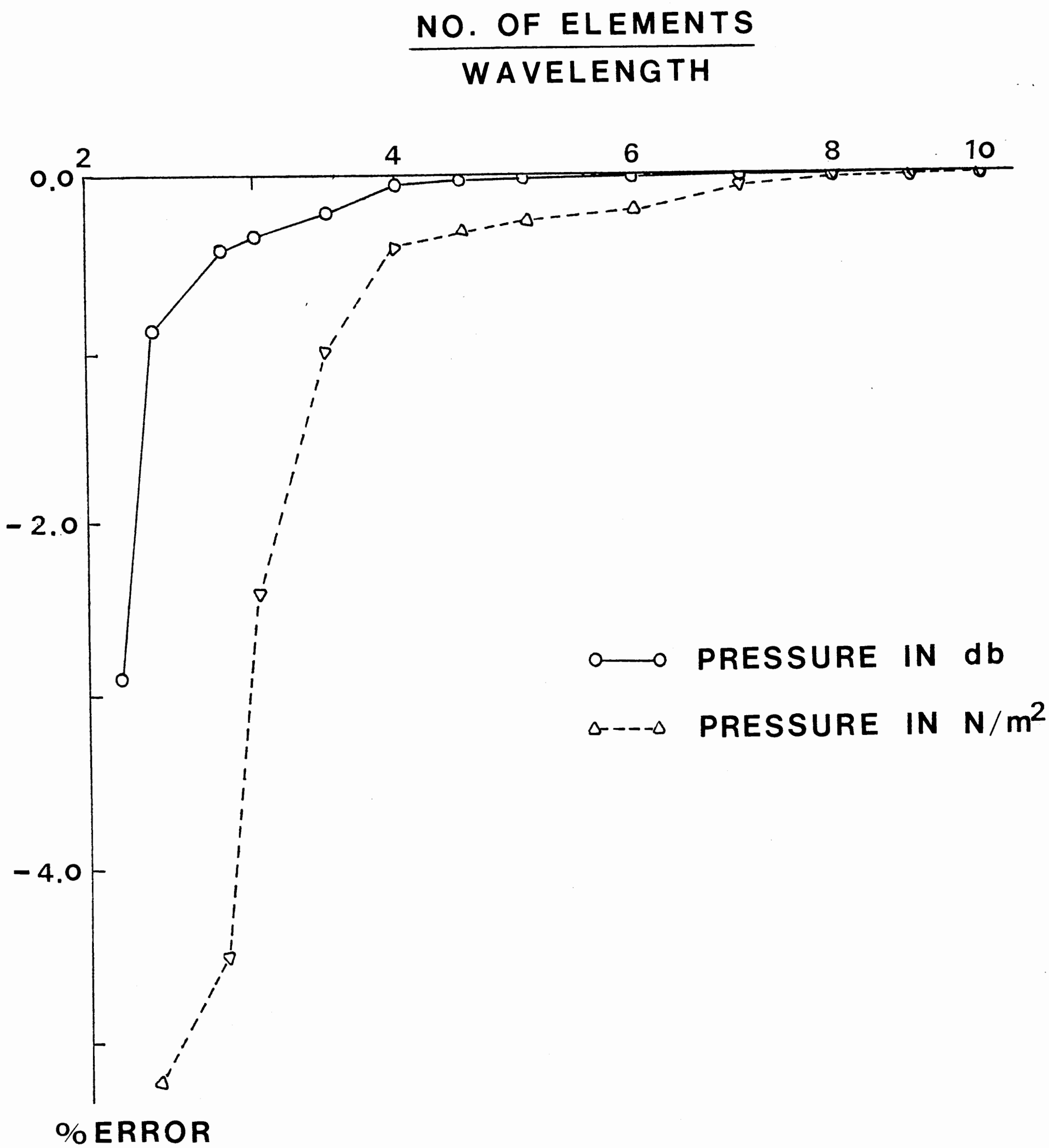


Figure 4. Convergence of Quadratic Isoparametric Element Solution to Acoustic Problem with no Airflow

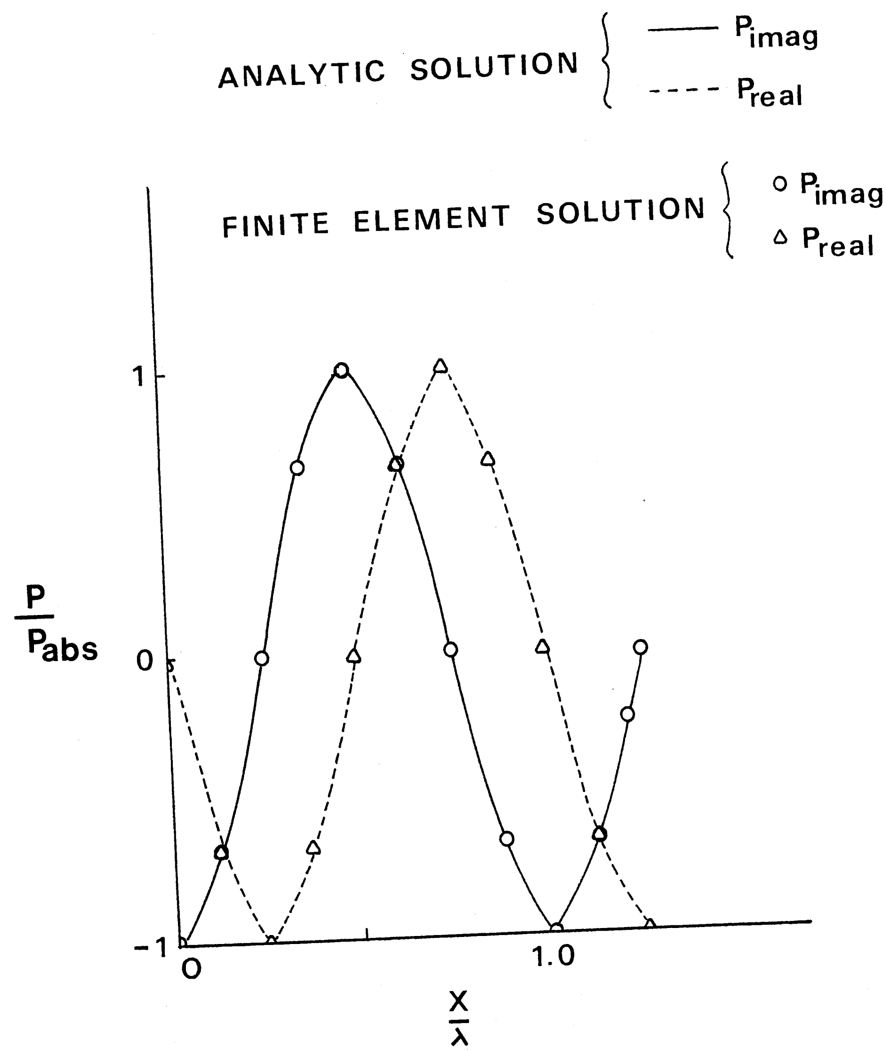


Figure 5. Comparison between Finite Element Solutions and Analytic Solutions with no Airflow

though the cubic isoparametric elements give slightly more accurate solutions, they need much larger bandwidth for the computation of the resultant linear algebraic system of equations. Thus, the quadratic isoparametric element is better for this problem.

When a plane wave is used as a input wave, the wave propagates along a straight duct in the shape of a plane wave as shown in Figure 6.

In order to consider the propagation of a higher mode input wave the concept of cut-off frequency should be introduced. The cut-off frequency is found from the equation

$$\nabla^2 P + k^2 P = 0 \tag{3.3}$$

Using the separation of variable method to Equation (3.3), we obtain

$$P(r,\theta,z) = \overline{P}\, J_m(k_{mn})\, e^{im\theta}\, e^{ik_x x} \tag{3.4}$$

where $\overline{P}$ is the amplitude of pressure and

$$k_x = \sqrt{k^2 - k_{mn}{}^2} \tag{3.5}$$

Applying the following boundary condition to Equation (3.4)

$$\frac{\partial P}{\partial n} = 0 \qquad \text{at } r = a \tag{3.6}$$

yields

$$J_m{}'(k_{mn}a) = 0 \tag{3.7}$$

Thus, the cut-off frequency is obtained form Equation (3.7). For instance, the cut-off frequency for a mode (1.0) inside a circular duct with a radius of 0.2m is 502.55 cycles/sec.

The higher mode (1.0) is treated here so that higher and lower

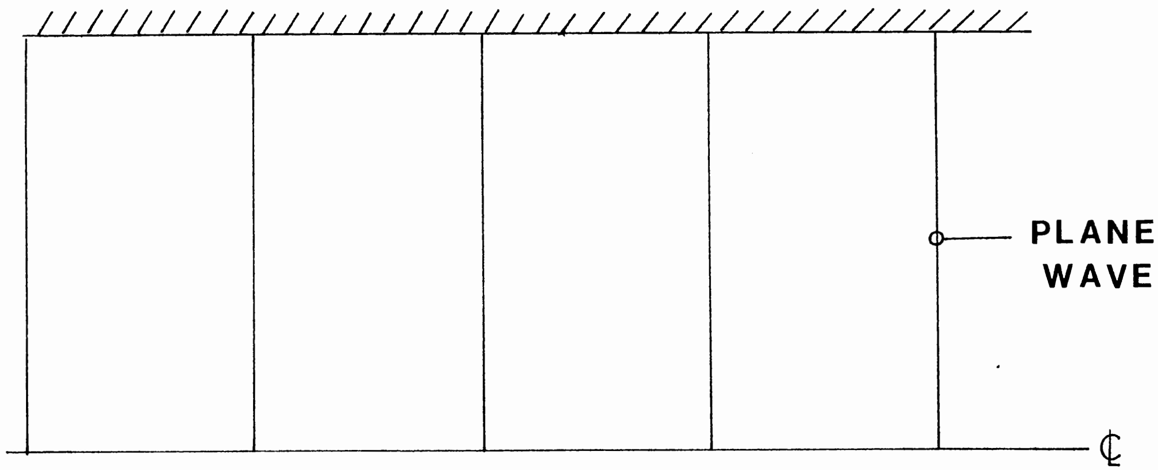


Figure 6. Acoustic Pressure Distribution Across Duct During Plane Wave Propagation

frequencies than the cut-off frequency are used as input to the duct. For higher mode (m,n) the anechoic termination impedence is $\rho c/\alpha mn$, and αmn is computed from

$$\alpha_{mn} = \sqrt{1 - \left(\frac{k_{mn}}{k}\right)^2} \tag{3.8}$$

which was given in Reference [24].

As expected the input wave with higher frequency than the cut-off frequency propagates without the acoustic pressure drop, but the input wave at lower frequency than the cut-off frequency propagates with the constant decaying rate in logarithmic scale of absolute pressure. The decaying rate is computed as below.

From Equations (3.5) and (3.8), we obtain

$$k_x = k\alpha_{mn} \tag{3.9}$$

and substituting Equation (3.9) into Equation (3.4) yields

$$P = P' \, e^{ik\alpha_{mn} x} \tag{3.10}$$

where

$$P' = \overline{P} \, J_m(k_{mn} r) \, e^{im\theta} \tag{3.11}$$

Thus, when the input frequency is less than the cut-off frequency at higher order mode, we get from Equation (3.8)

$$\alpha_{mn} = i\sqrt{\left(\frac{k_{mn}}{k}\right)^2 - 1} = i\beta_{mn} \tag{3.12}$$

Substituting Equation (3.12) into Equation (3.10) gives

$$P = P' \, e^{-k\beta_{mn} x} \tag{3.13}$$

Consequently, the decaying rate in logarithmic scale is $-k\beta_{mn}$, and the input wave with lower frequency has the larger decaying rate. Figure 7 shows the phenomena mentioned above.

Sound Field In Varying Cross-section Ducts

Because the accuracy of the finite element solution was proved for the sound field in a constant area duct with no flow in last section, the accuracy of the finite element solution for the sound in a varying cross-section duct with no flow will be verified. As that example problem, a bottle-like duct with a closed end is studied using the finite element formulation. The piston-type sound source is used at the bottle mouth and the hard walls exist in the other sides.

When Ka is larger than 1.3, the higher modes become significant so that it is hard to get the exact solution using the analytical approach. Therefore, the finite element formulation is used to solve the bottle-like duct with a closed end when Ka is equal to 1.95 which is larger than 1.3. The finite element solution and the experimental result are shown in Figure 8. The experimental result was given in Reference [18]. The reason there is a slight difference between both results is that the duct shape obtained by curve fitting using two-dimensional quadratic isoparametric elements is not the actual duct shape used in experimental analysis. The finite element mesh used for this problem is shown in Figure 9, and this mesh was chosen based on the convergence curve given in the last section.

In last and this sections, the accuracy of the finite element solution was proved for the duct acoustic problem not including flow in a duct.

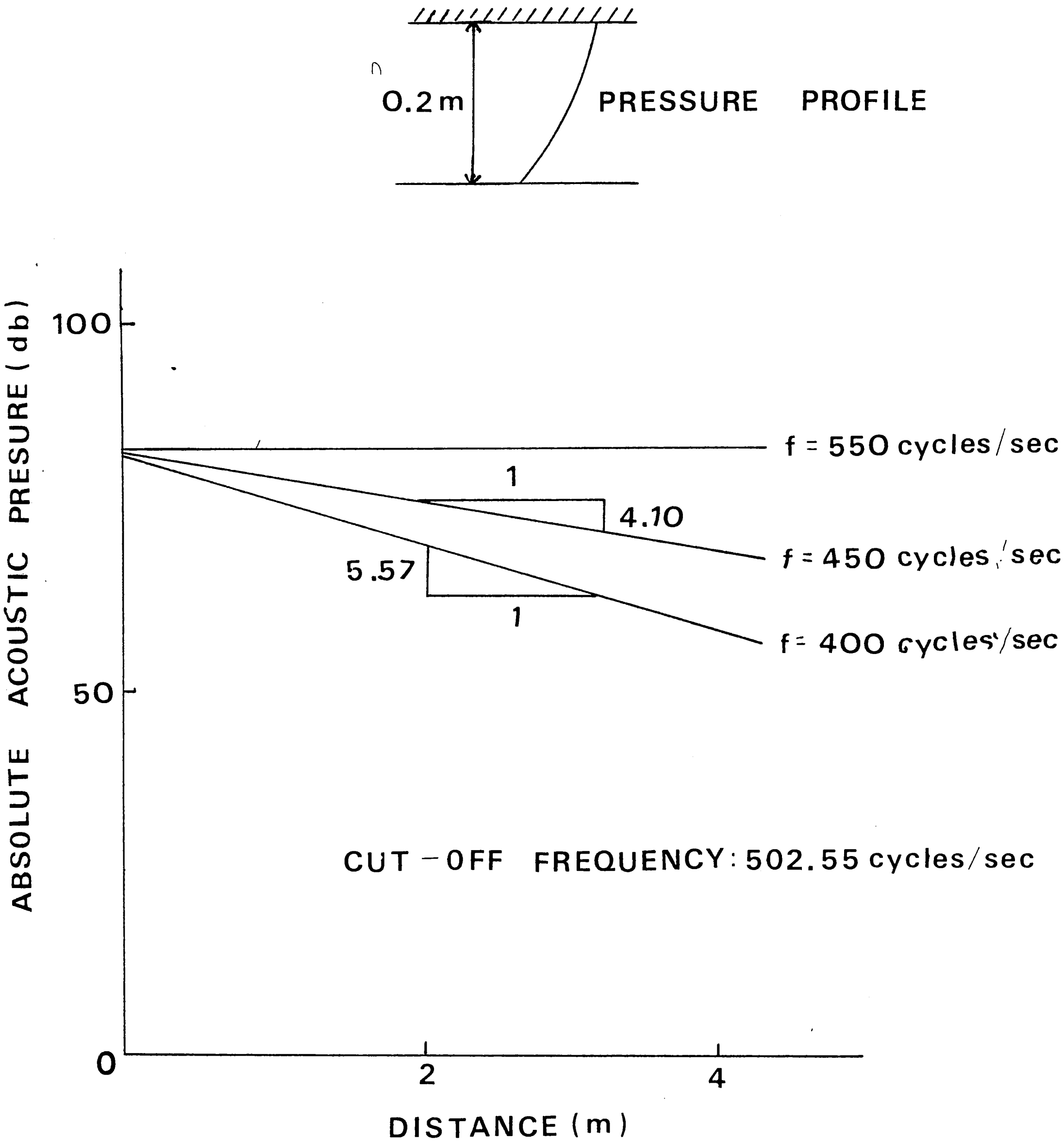


Figure 7. Characteristic of Cut-off Frequency for Mode (1,0)

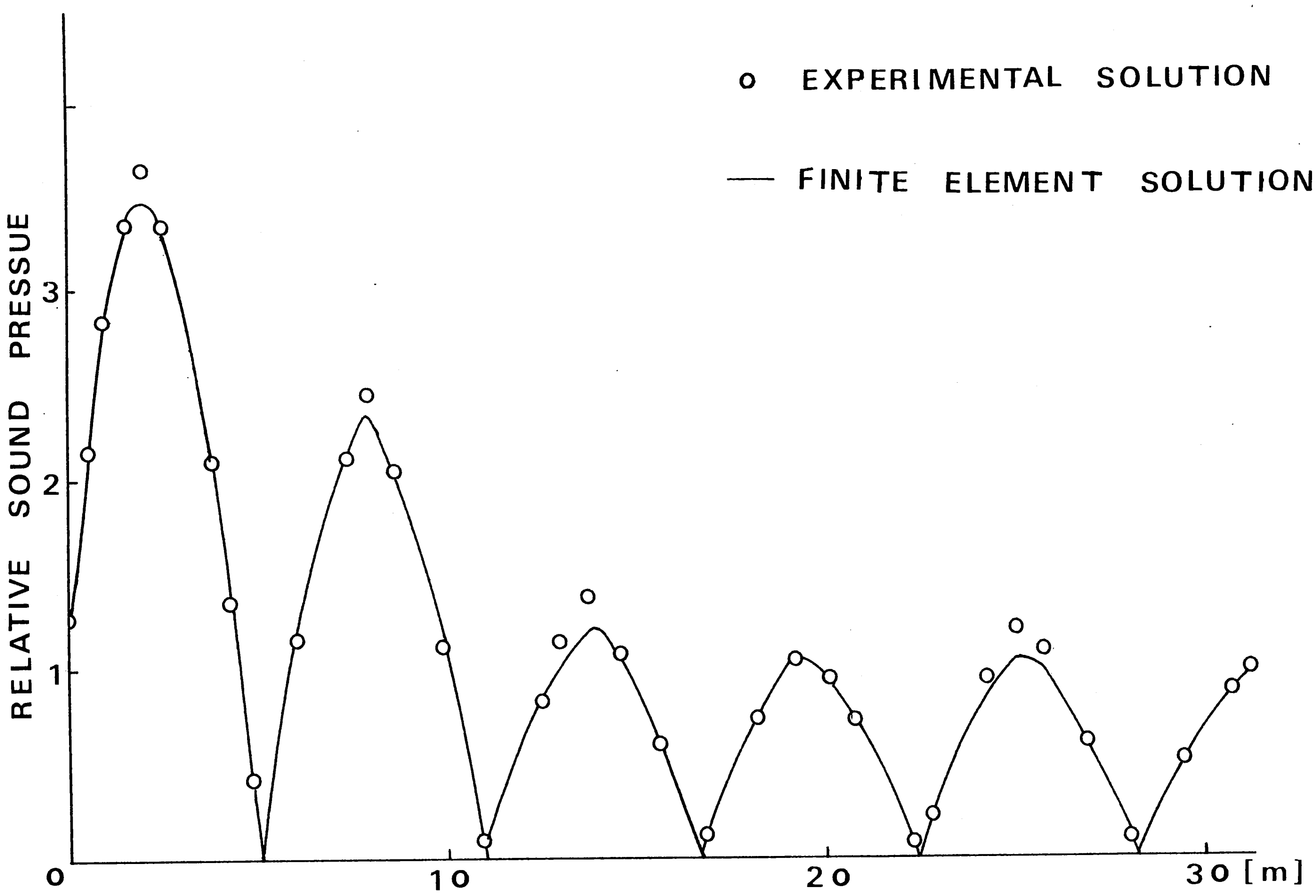


Figure 8. Sound Field in a Rottle-like Duct (ka = 1.95)

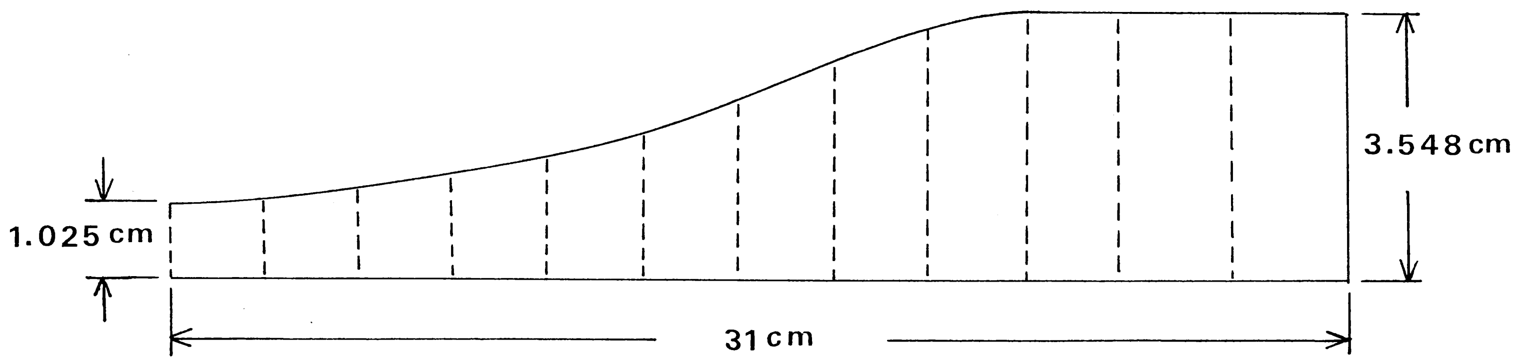


Figure 9. Finite Element Mesh for a Rottle-like Duct

Sound Field In Circular Straight Ducts

With Uniform Flow

When we consider the uniform airflow in a duct, the terms containing a velocity V in radial direction and a derivatives with respect to y can be droped from Equation (2.1). Also, if the plane wave is treated, the equation is simplified to the form

$$(1 - \frac{U^2}{c^2}) \frac{\partial^2 P}{\partial x^2} - i \frac{2U}{c^2} \frac{\partial P}{\partial x} + k^2 P = 0 \qquad (3.14)$$

The analytic solution to Equation (3.14) with proper boundary conditions is disscussed in Appendix B.

The convergence curves for the constant area duct problem including uniform flow are given in Figures 10 and 11. Figure 10 is for the case of flow along the sound propagation, and Figure 11 is for the case of flow against the sound propagation. The number of elements per wavelength can be selected based on these convergence curves for the constant area duct with uniform flow.

The comparisons between the convergence curves for the quadratic isoparametric elements shown in Figures 10 and 11 and those for the cubic isoparametric elements given in Reference [18] indicate that the quadratic isoparametric elements are better even though the cubic isoparametric elements give slightly more accurate results because of the same reason as mentioned for the convergence curve of the no flow case.

The flow in the same direction as the sound propagation convects the sound propagation so that a higher acoustic pressure propagates along a duct. The comparing curves between finite element and exact solutions are given for M=0.5 and M=-0.5 respectively in Figures 12 and

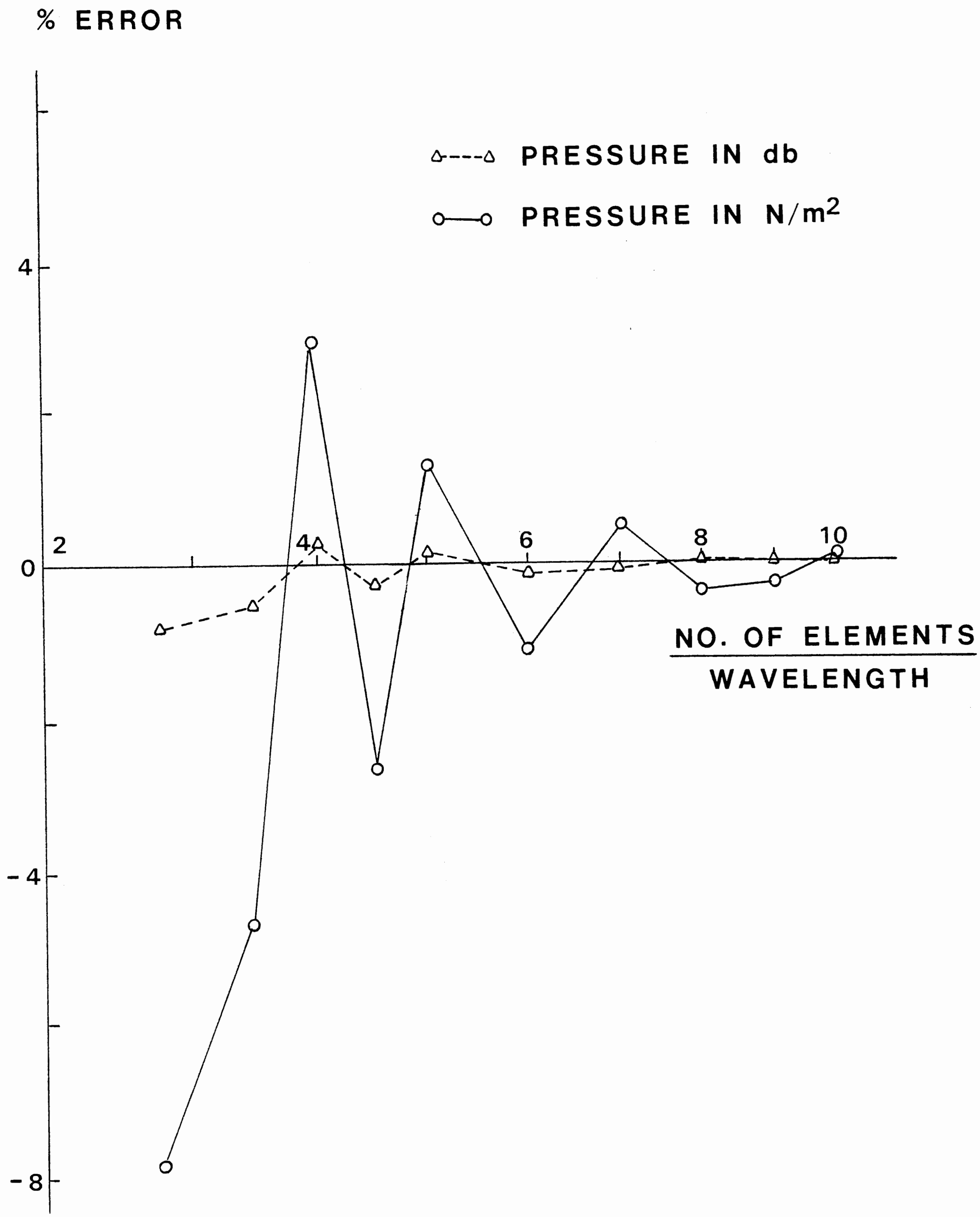


Figure 10. Convergence of Quadratic Isoparametric Element Solution to Acoustic Problem with Airflow of M = 0.5

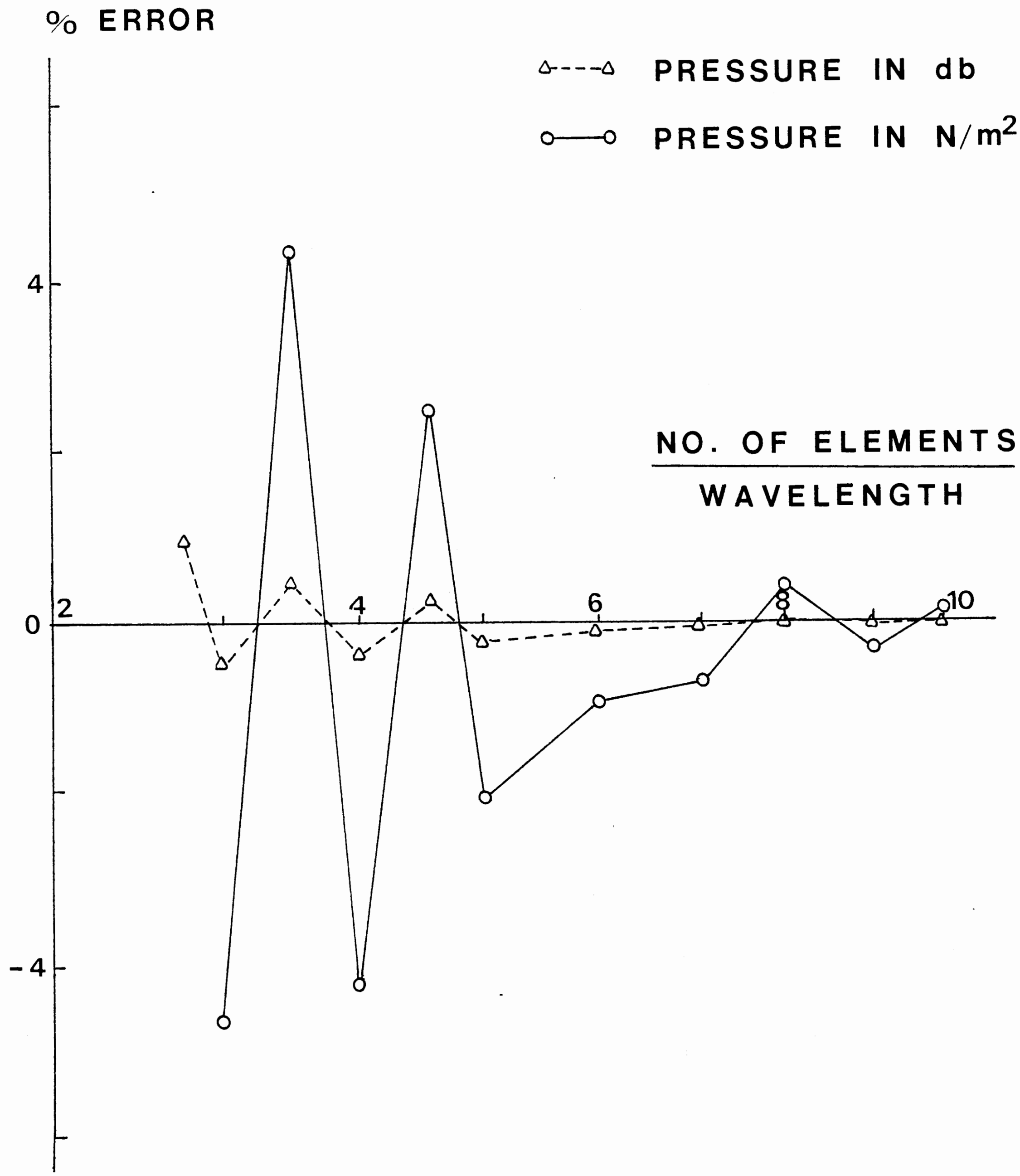


Figure 11. Convergence of Quadratic Isoparametric Element Solution to Acoustic Problem with Airflow of M = 0.5

13. The finite element solutions are in very good agreement with the analytic solutions developed in Appendix B. In Figures 12 and 13 λ is the wavelength of the convected sound when the flow exists inside a duct and λ is equal to $(1+M)\lambda o$, where λo is the wavelength when no flow exists in a duct.

From this study it can be concluded that the finite element analysis gives the accurate solutions for the constant area duct acoustic problems including uniform flow.

Sound Field In Circular Straight Ducts With Constant Gradient Flow

The circular straight duct containing constant gradient flow as shown in Figure 14 is analyzed using the finite element method. Even if the plane wave is used as an input wave, the wave inside a duct is not the plane wave any more because of the refraction and the convection due to the shear flow. It is very difficult to predict the anechoic termination impedence so that a long duct can be used to attenuate the reflected wave due to the disharmony of impedence at the right-side boundary. Three ducts with different lengths but same radii are used to find a duct whose front portion is not affected by the reflected wave due to the disharmony of the anechoic termination impedence. To do so the pressure distributions along the radii of the front portions of three different ducts with same input frequencies are investigated. They are shown in Figure 15. As shown in that figure the pressure distributions for two ducts with 20cm and 30cm lengths are almost same. This means that 20cm of length is enough to decay the reflected wave coming from the right-side boundary so that the front portion of the duct may not

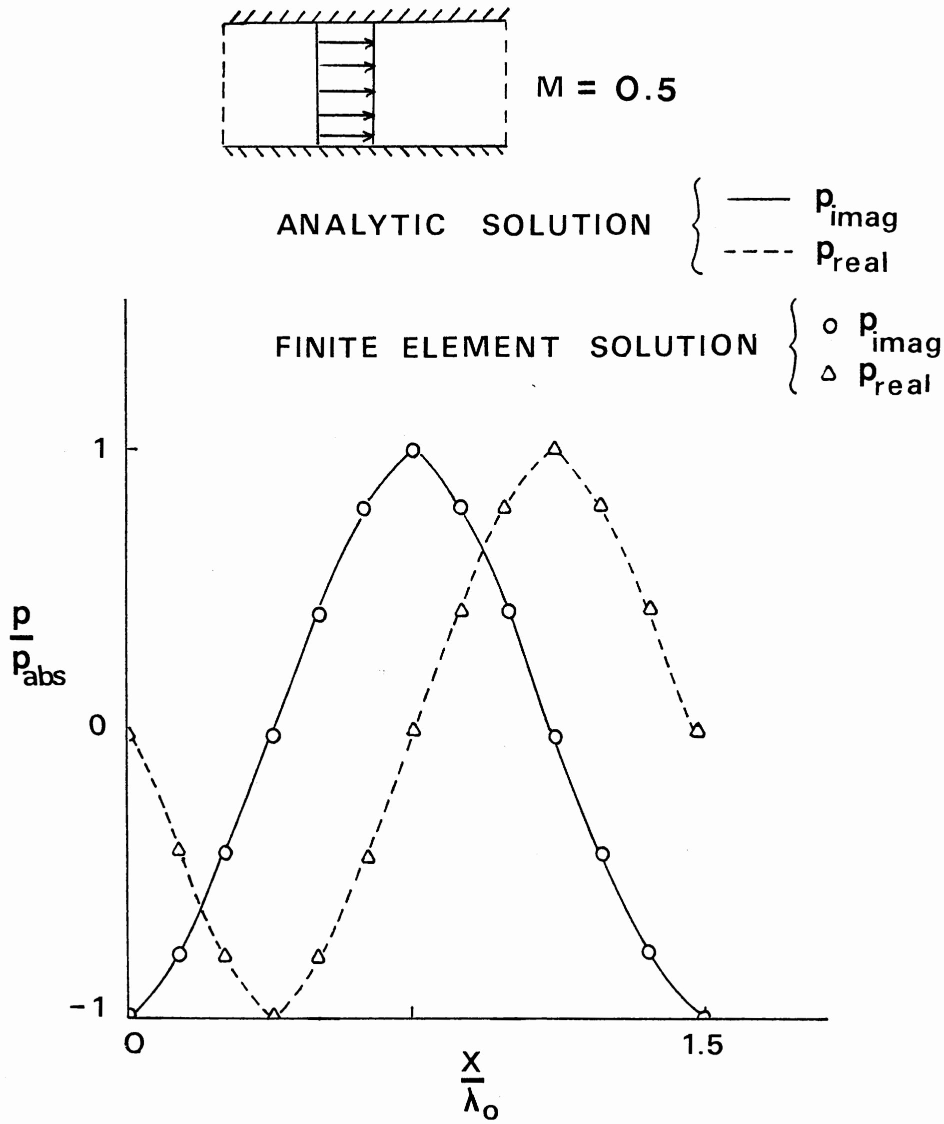


Figure 12. Comparison between Finite Element and Analytic Solutions for M = +0.5

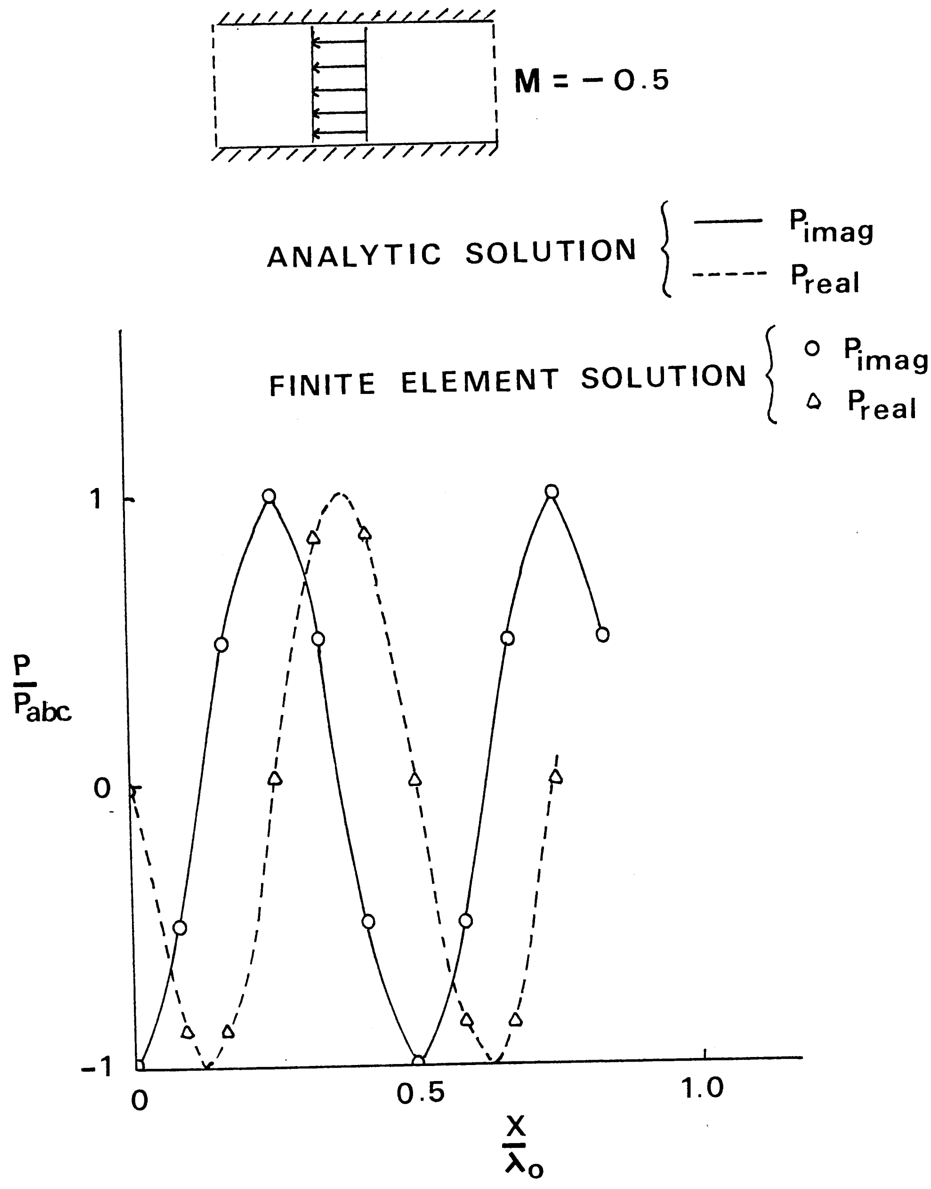


Figure 13. Comparison between Finite Element and Analytic Solutions for M = -0.5

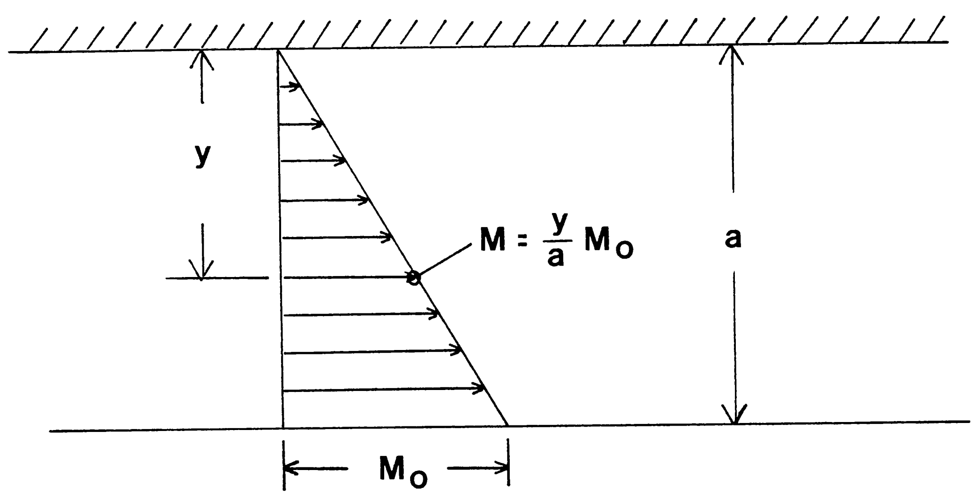


Figure 14. Constant Gradient Velocity
Inside a Duct

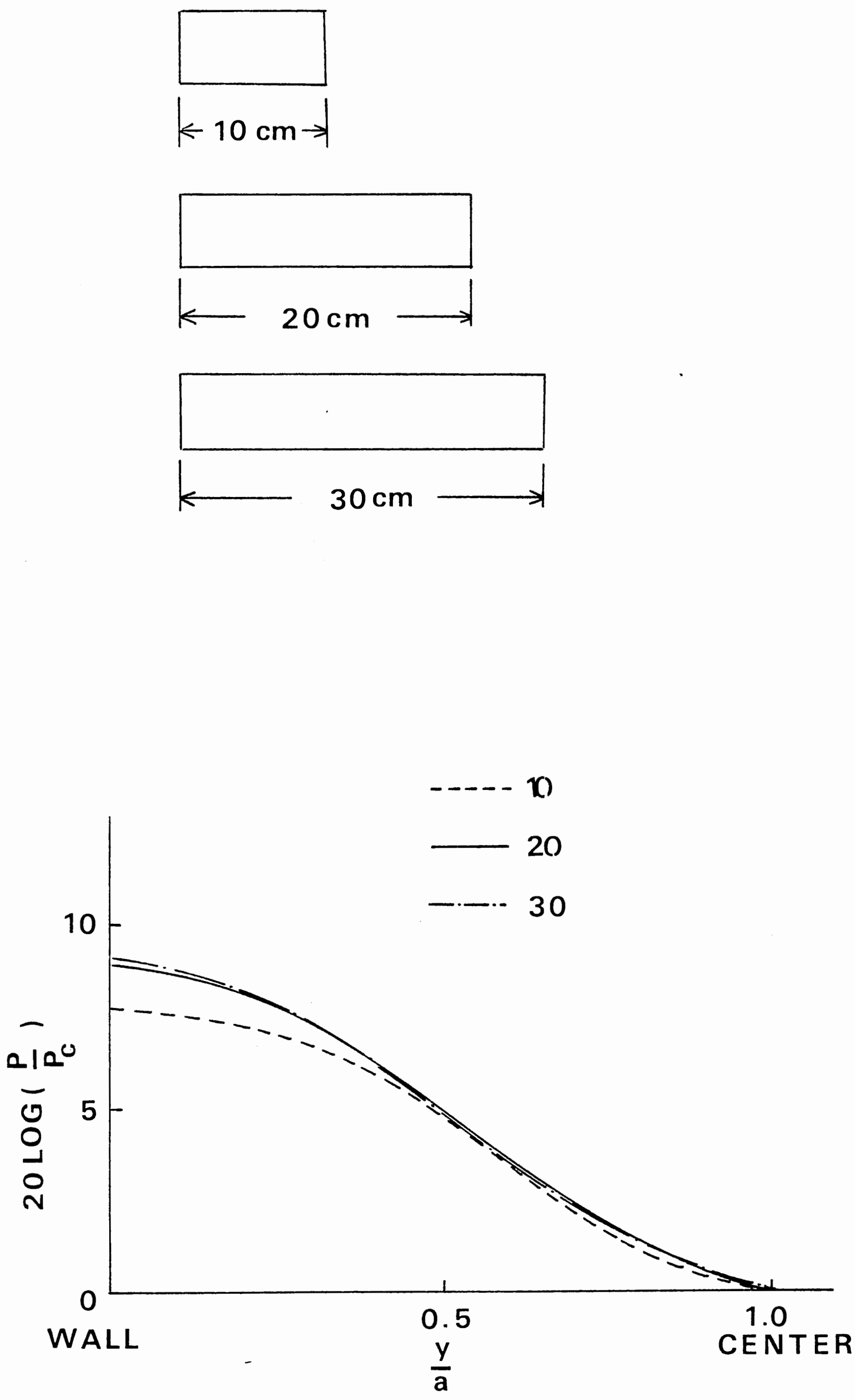


Figure 15. Acoustic Pressure Distributions Across Ducts with Different Lengths

be influenced by the reflected wave. Thus, this length duct is used for this study.

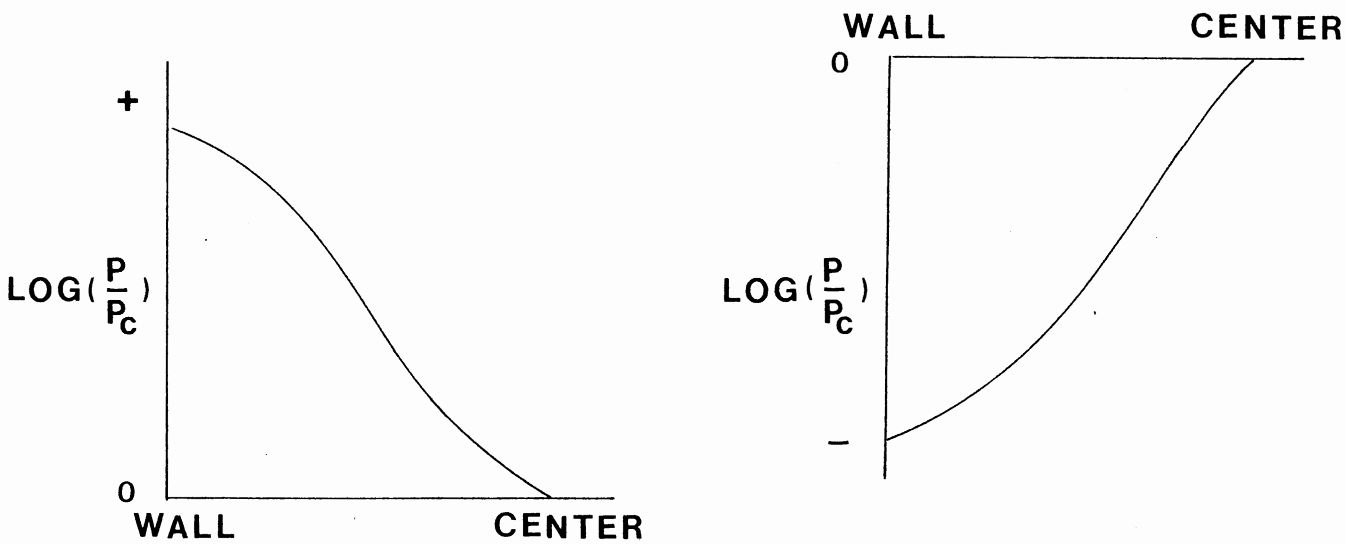
Using the finite element analysis, two kinds of mode shpaes are obtained for the pressure distribution across a duct, which are shown in Figure 16. In the first mode shape, the sound is directed into the boundary walls due to the velocity gradient in shear flow. This was predicted using the analytic study in References [19,25]. Moreover, the large value of velocity of shear flow makes a greater change of pressure across a duct than the small value of velocity, which is shown in Figure 17. This result was also predicted by Pridmore-Brown [19].

The above phenomena can be used in reducing undesired sound by making it pass through an acoustically lined duct, and we can conclude the finite element analysis combined with the convected wave equation can be used to solve the duct acoustic problem including shear flow.

Sound Propagation Inside Circular Converging-Diverging Duct With Two Dimensional Flow

The converging-diverging duct with two-dimensional flow is analyzed using the finite element method. As the geometry of the duct becomes complicated, the phenomenon of sound propagation in a duct also becomes difficult to predict. Even though the plane wave enters the inlet of a converging-diverging duct, the plane wave does not persist any more along the duct. Particuarly, with the existence of flow inside a duct, the acoustic pressure field comes to be more complex. Figure 18 shows the finite element mesh to be solved.

First of all, the acoustic field is studied without considering air-flow in a converging-diverging duct. As the plane wave propagates



(a) First Mode Shape (b) Second Mode Shape

Figure 16. Two Mode Shapes for Pressure Distribution Across a Duct Containing Shear Flow

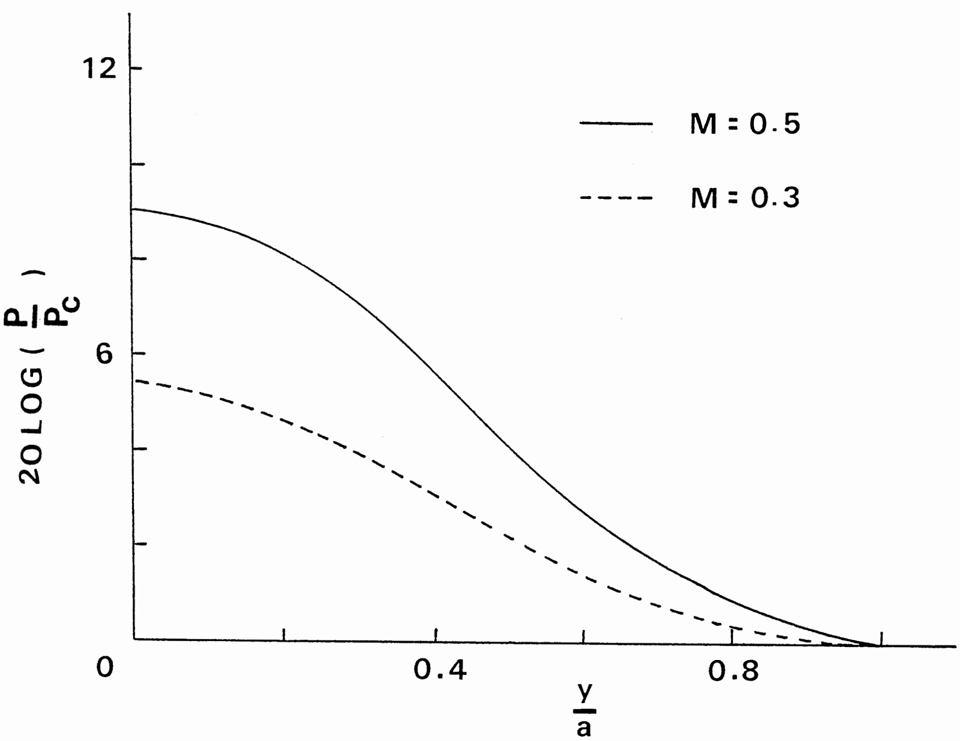


Figure 17. Acoustic Pressure Distribution Across Duct for Different Velocities of Airflow and Same Frequency

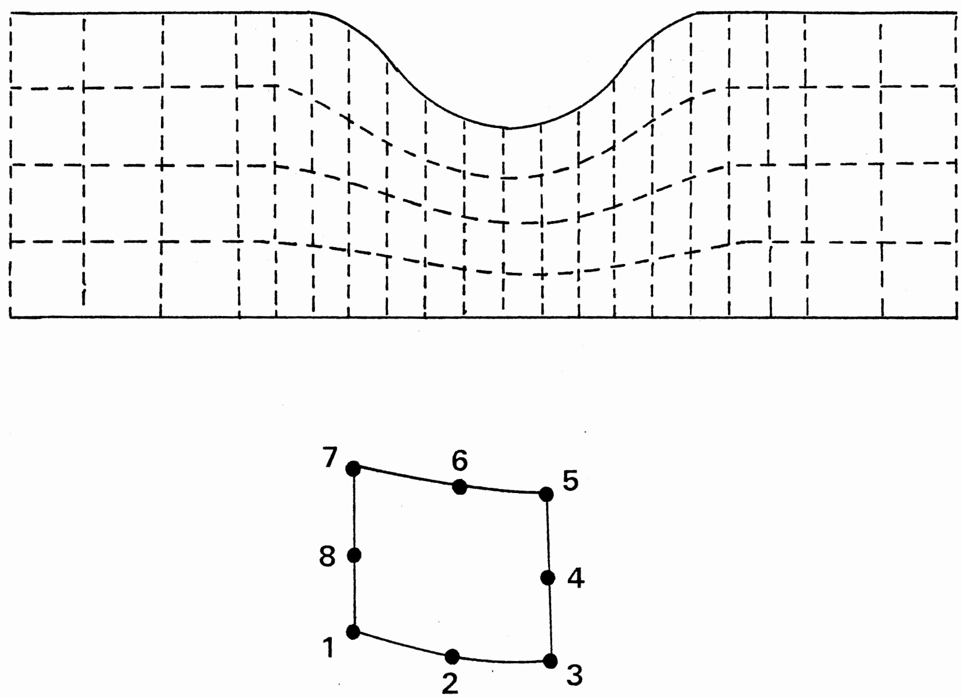


Figure 18. Finite Element for Convergent-Divergent Duct

inside a duct, the wave is in the shpae of standing wave in pre-nozzle and nozzle sections. However, in post-nozzle section there is almost no variation in acoustic pressure as shown in Figures 19 and 20. The wave almost remains a plane wave in pre-nozzle section, but the wave is converted into higher mode in nozzle seciton. After the nozzle section the higher mode decays due to the cut-off frequency characteristics. Thus, in this section the shape of wave comes to resemble the plane wave as the sound continues to propagate along a duct, which is shown in Figure 21.

Next, we assume the uniform flow at the inlet of a converging-diverging duct. Even if the inlet velocity is assumed to be uniflow, the velocity distribution inside a converging-diverging duct does not remain uniform throughout the duct. A host of papers have been published about the velocity field in a varying area duct [26-30]. In this paper, the finite element method using the potential flow theory is used to compute the velocities in axial and radial directions of a converging-diverging duct. The obtained velocity distribution in a converging-diverging duct is shown in Figure 22. Here, the length of arrow indicates the relative magnitude of velocity. As clearly shown in the figure, the radial velocity component cannot be neglected around the nozzle section. Thus, the terms containing the radial velocity in the wave equation cannot be also omitted. The sound propagation with such velocity component is solved and also shown in Figures 19 and 20. The characteristics of sound propagation is preserved with the flow, but the peak of acoustic pressure is different and there is also phase shift between no flow and flow cases. Moreover, in case of Ka=2.2 the acoustic pressure peak carrying flow is higher than that not carrying flow, but in case of

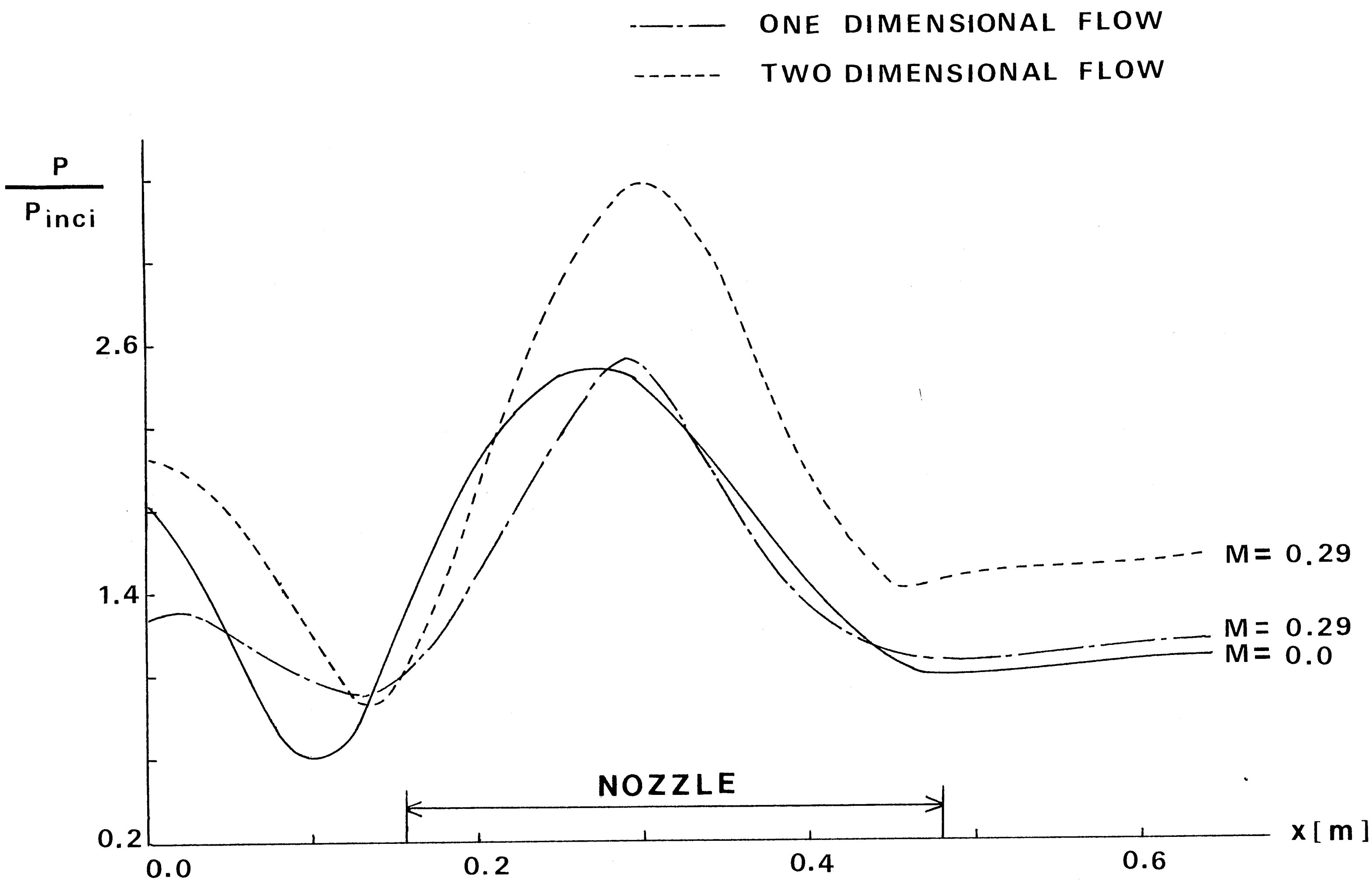


Figure 19. Acoustic Pressure Field Inside Converging-Diverging Duct with and without Airflow (ka = 2.2)

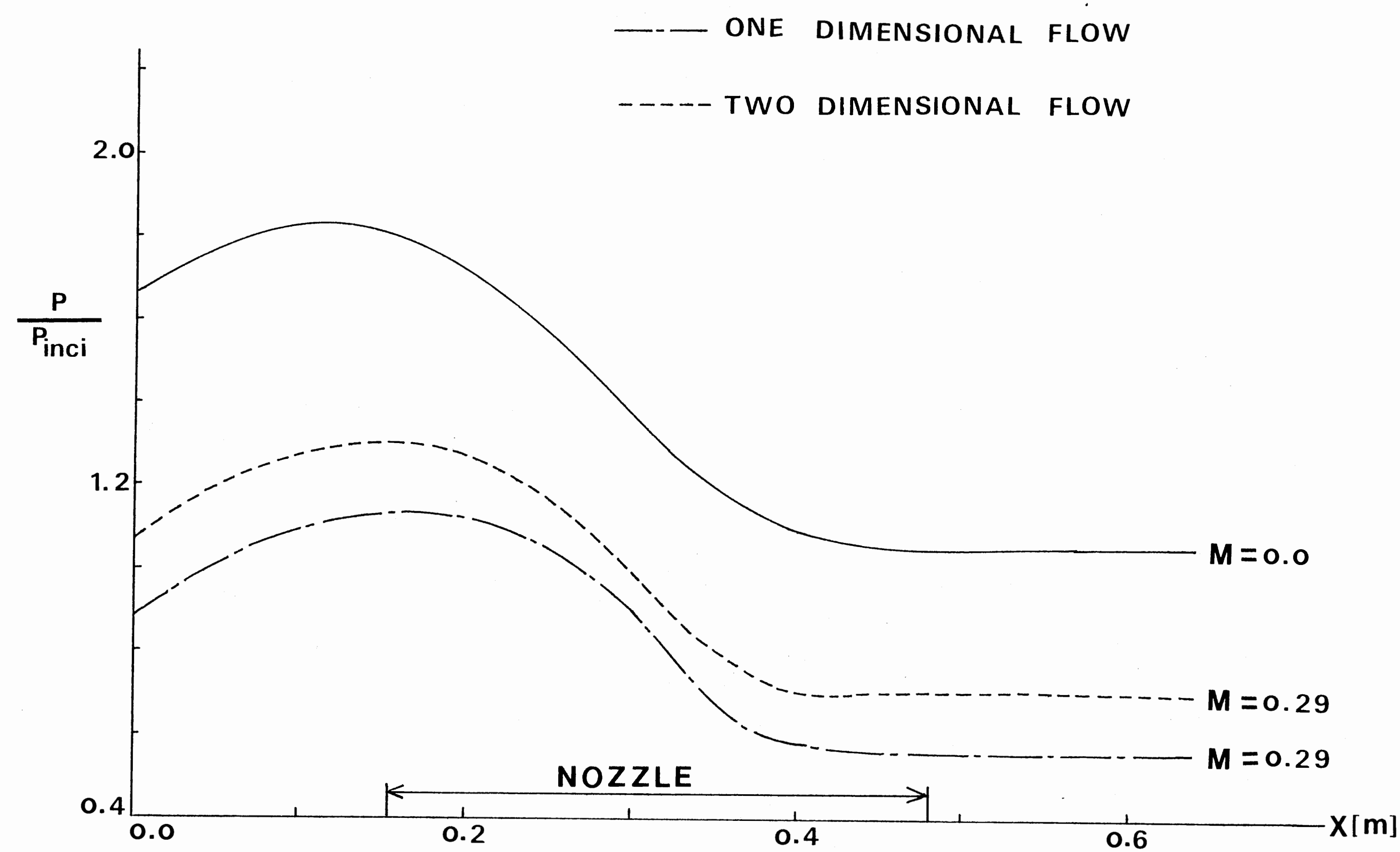


Figure 20. Acoustic Pressure Field Inside Converging-Diverging Duct with and without Airflow (ka = 0.73)

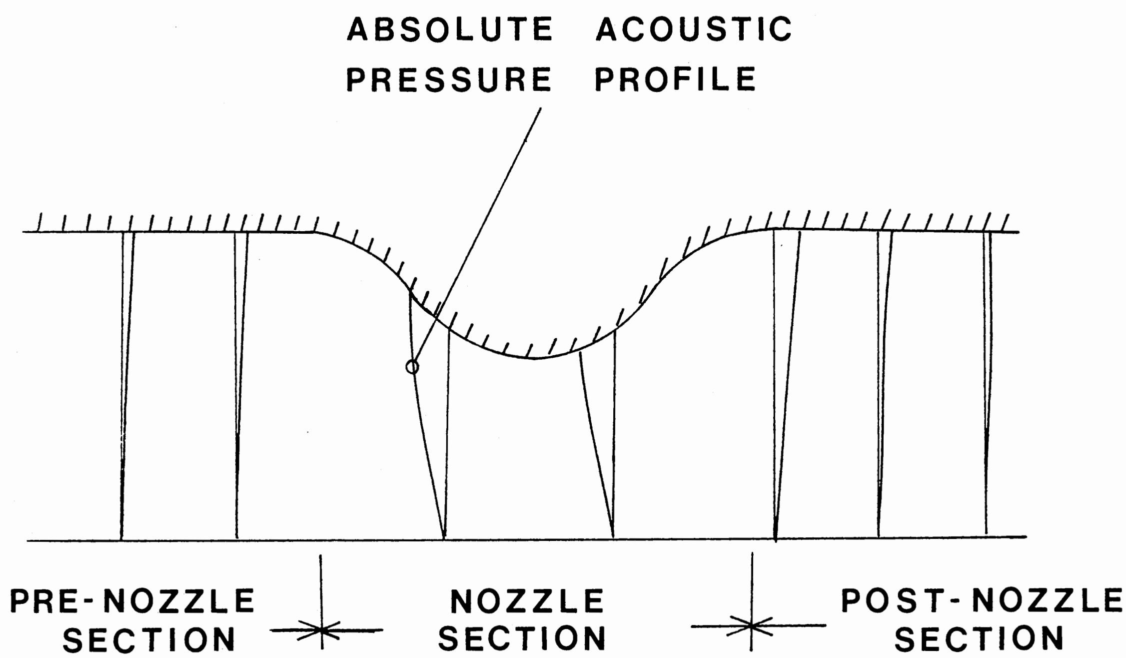


Figure 21. Acoustic Pressure Profile Across Converging-Diverging Duct with no Airflow

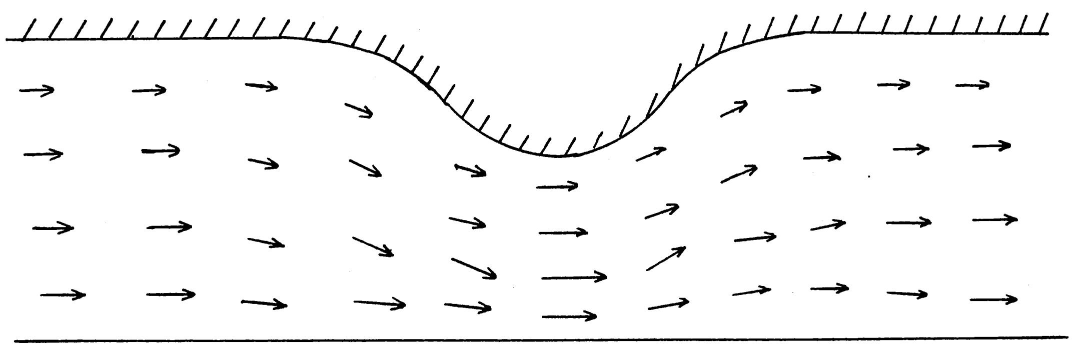


Figure 22. Velocity Distribution Inside a Converging-Diverging Duct

Ka=0.73 the above statement is reversed. These are different results compared to the results obtained using only one-dimensional flow which does not include the radial velocities. The finite element analysis for these problem using one-dimensional flow were also shown in Figures 19 and 20. Thus, the radial velocity terms cannot be neglected in solving the converging-diverging duct acoustic problem containing flow in the duct.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Some important conclusions can be deduced through this study as mentioned below:

1. The Galerkin finite element method in combination with two-dimensional isoparametric element was proved to be powerful in solving duct acoustic problems. In particular, this method was very useful to the duct with complex geometry of boundaries and complex boundary conditions. This kind of problems are very hard to solve by analytic and other approximate methods.

2. The duct acoustic problems with shear flow in a duct were verified to be solved using the convected wave equation. The finite element analysis using the convected wave equation gave reasonable results as predicted by analytical method. Moreover, two different mode shapes were obtained using the finite element analysis. The first mode of two modes was predicted by some authors [19,25].

3. Two-dimensional quadratric isoparametric element was found to have advantages compared to linear or cubic isoparametric elements. The linear isoparametric element connot model the curved boundary, and there is no big difference in convergence curve between quadratic and cubic isoparametric elements. Thus, it can be concluded that the quadratic isoparametric element is the best for the duct acoustic problems.

4. The result of sound propagation in a converging-diverging duct

containing two-dimensional flow was quite different from that with one-dimensional flow. Hence, the vertical velocity cannot be neglected in predicting the sound propagation in a converging-diverging duct containing flow.

Next, some works can be recommended for the further research based on this study.

1. As mentioned before, the basic domain equation used throughout this research was derived based on the assumption that the velocity of flow in an element was constant. This assumption is reasonable when the smaller and smaller element mesh is used. However, if Equation (1.7) is used as a domain equation, it need not be assumed that the velocity is constant in an element. Instead, the larger finite element mesh can be used and the finite element solution will be more accurate for the duct acoustic problems including diverse flow. The disadvantage of Equation (1.7) is that the higher order shape function is required because of C1 conformability.

2. Instead of an axisymmetric duct, any shape of duct can be analyzed using the finite element method. The different shape of duct requires some change in numerical integration procedure. In this case, Equations (2.43) and (2.44) cannot be used.

The further investigation as mentioned above will make the finite element method be used to analyze more general duct acoustic problems.

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APPENDIX A

BOUNDARY CONDITION

The momentum equation in the x-direction is

$$\rho \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} \qquad (A.1)$$

Here, the velocity can be expressed as

$$u = i\omega(A \cos(\omega t-kx) + B \cos(\omega t+kx)) \qquad (A.2)$$

where A and B are arbitrary constants.

Substituting Equation (A.2) into Equation (A.1) yields

$$i\rho\omega u + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} \qquad (A.3)$$

At the right-hand side boundary of a duct, u can be expressed as

$$u = \frac{P}{z} \qquad (A.4)$$

Substituting Equation (A.4) into (A.3) gives

$$i\rho\omega(\frac{P}{z}) + \rho U \frac{\partial}{\partial x} (\frac{P}{z}) + \rho V \frac{\partial}{\partial y} (\frac{P}{z}) = - \frac{\partial P}{\partial x} \qquad (A.5)$$

Using the assumption that the impedence is independent of space, we get

$$i\rho\omega \frac{P}{z} + \frac{U}{z}\frac{\partial P}{\partial x} + \frac{V}{z}\frac{\partial P}{\partial y} = -\frac{\partial P}{\partial x} = -\frac{\partial P}{\partial n} \qquad (A.6)$$

The boundary conditions for the other side walls can be deduced in a similar way.

APPENDIX B

ANALYTIC SOLUTION FOR A STRAIGHT DUCT ACOUSTIC PROBLEM WITH UNIFORM FLOW

The domain equation for this case is

$$(1 - M^2) \frac{\partial^2 P}{\partial x^2} - 2i\omega \frac{M}{c} \frac{\partial P}{\partial x} + k^2 P = 0 \tag{B.1}$$

The solution for Equation (B.1) is

$$P = A\, e^{-i\frac{kx}{1+M}} + B\, e^{i\frac{kx}{1-M}} \tag{B.2}$$

The corresponding boundary conditions are

$$\frac{\partial P}{\partial n} = - \frac{\partial P}{\partial x} = f(x,y) \qquad \text{at } x = 0 \tag{B.3}$$

and

$$- \frac{\partial P}{\partial n} = - \frac{\partial P}{\partial x} = i\rho\omega \frac{P}{z} + \frac{U}{z} \frac{\partial P}{\partial x} \qquad \text{at } x = L \tag{B.4}$$

where L is the length of a straight circular duct.

When only right-traveling wave exists, the impedence is

$$z = \rho c \tag{B.5}$$

When only left-traveling wave exists, the impedence is

$$z = - \rho c \tag{B.6}$$

Thus, when only right-traveling wave exists, substituting Equation (B.2), its derivative with respect to x and Equation (B.5) into Equation (B.4) yields B=0 so that we obtain

$$P = A\, e^{-i \frac{kx}{1+M}} \tag{B.7}$$

Next, from Equations (B.3) and (B.7) the final solution is

$$P = - i \frac{1+M}{k} f(x,y)\, e^{-i \frac{kx}{1+M}} \tag{B.8}$$

Hence, the real and imaginary parts of pressure can be obtained from Equation (B.8).

When only left-traveling wave exists, the similar procedure can be applied to obtain the analytic solution.

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